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COMPOSITE ROBUST ACTIVE CONTROL OF SEISMICALLY EXCITED STRUCTURES WITH ACTUATOR DYNAMICS

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SUMMARY

In this paper, composite robust active control schemes are proposed for a class of non-linear base isolated structures in the presence of unknown seismic excitation, parametrical uncertainties and actuator dynamics. Only the information on state variables of the structural base and the first floor of the main structure has been used in the control design. A numerical simulation example is given for a ten-storeyed base isolated structure under the *El Centro* earthquake to show the effectiveness of the proposed control scheme. © 1998 John Wiley & Sons, Ltd.

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KEY WORDS: active control; composite control; base isolated structures; actuator dynamics

1. INTRODUCTION

The idea of co-operatively combining both passive and active control systems has been increasingly considered in the last years in the context of vibration mitigation of civil engineering structures. For seismic protection of buildings, combinations of (passive) base isolators and (active) feedback controllers applying forces to the base have been proposed. Since those active forces react to the absolute motion, they are able to supply an additional resistant scheme not attainable by purely passive means when the structure is under ground excitation.

Within the above scheme, recent works have approached the problem of designing active controllers through the understanding of the interaction between the base isolation system and the structure by considering both as two coupled systems.[1–5] In these works the objective has been to ensure a form of stability of the overall system resulting in a significant reduction of the motion of both the structure and the base. Important issues of this approach are that the system parameters and the excitation do not need to be known (only their bounds) and the base isolation may behave non-linearly, thus, resulting in a robust control scheme. This paper follows the same direction, but it takes into account a new element: the actuator dynamics. The importance of the interaction of the actuator producing the active forces with the structure has been recognized in the literature and ways to account for have been proposed.[6–8]

This paper deals with the problem of designing a robust active controller for a class of seismically excited base isolated structures which include the effects of a actuator dynamics and parametrical uncertainties. The dynamical behaviour of the system is described by a mathematical model composed of three coupled subsystems: the main structure subsystem and the base isolation subsystem, which are described by a set of

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second-order differential equations, together with the actuator subsystem, which is described by a first-order differential equation. It is assumed that the system parameters (mass, damping, stiffness) are of unknown values but with known upper bounds and the seismic excitation is an unknown disturbance. In the paper, control design is first done in accordance with the composite control strategy. Then, stability analysis is made based upon Lyapunov stability theory. Finally, a numerical simulation example is given for a ten storeyed base isolated structure under the *El Centro* earthquake to show the effectiveness of the proposed robust control scheme.

2. PROBLEM DESCRIPTION

Consider an active controlled non-linear base isolated structure with a hydraulic actuator, whose dynamic behaviour is described by the following model composed of three coupled subsystems:

Main structure

$$\mathbf{M}\ddot{\mathbf{q}}_{\mathrm{r}} + \mathbf{C}\dot{\mathbf{q}}_{\mathrm{r}} + \mathbf{K}\mathbf{q}_{\mathrm{r}} = \boldsymbol{\rho}(q_{\mathrm{c}}, \dot{q}_{\mathrm{c}}) \tag{1a}$$

$$\boldsymbol{\rho}(q_{\mathrm{c}}, \dot{q}_{\mathrm{c}}) =: \boldsymbol{\rho}_1 \dot{q}_{\mathrm{c}} + \boldsymbol{\rho}_2 q_{\mathrm{c}}, \quad \boldsymbol{\rho}_1 = [c_1, 0, \ldots, 0]^{\mathrm{T}}, \quad \boldsymbol{\rho}_2 = [k_1, 0, \ldots, 0]^{\mathrm{T}} \tag{1b}$$

Base isolation

$$m_0 \ddot{q}_{\mathrm{c}} + (c_0 + c_1)\dot{q}_{\mathrm{c}} + (k_0 + k_1) q_{\mathrm{c}} - c_1 \dot{q}_{\mathrm{r}1} - k_1 q_{\mathrm{r}1} + f(q_{\mathrm{c}}, \dot{q}_{\mathrm{c}}, d, \dot{d}) = v \tag{2a}$$

$$f(q_{\mathrm{c}}, \dot{q}_{\mathrm{c}}, d, \dot{d}) =: -c_0 \dot{d} - k_0 d + f_{\mathrm{N}}(q_{\mathrm{c}}, \dot{q}_{\mathrm{c}}, d, \dot{d}) \tag{2b}$$

Hydraulic actuator[8]

$$P_{\mathrm{v}} \dot{v} + P_{\ell} v + P_{\mathrm{a}} \dot{q}_{\mathrm{c}} = u, \quad P_{\mathrm{v}} =: \frac{C_{\mathrm{v}}}{4\beta P_{\mathrm{a}}} > 0, \quad P_{\ell} =: \frac{C_{\ell}}{P_{\mathrm{a}}} > 0, \quad P_{\mathrm{a}} > 0 \tag{3}$$

where $\mathbf{q}_{\mathrm{r}} = [q_{\mathrm{r}1}, q_{\mathrm{r}2}, \ldots, q_{\mathrm{r}n_{\mathrm{r}}}]^{\mathrm{T}} \in \mathbb{R}^{n_{\mathrm{r}}}$ and $q_{\mathrm{c}} \in \mathbb{R}$ represent the horizontal displacements of each floor and the structural base with respect to an inertial frame, respectively. The coupling effect between the main structure subsystem (1) and the base isolation subsystem (2) is described by a vector function $\boldsymbol{\rho}(q_{\mathrm{c}}, \dot{q}_{\mathrm{c}}) \in \mathbb{R}^{n_{\mathrm{r}}}$, as defined in equation (1b), $\mathbf{M}$, $\mathbf{C}$ and $\mathbf{K} \in \mathbb{R}^{n_{\mathrm{r}} \times n_{\mathrm{r}}}$ are positive-definite matrices corresponding to the mass, damping and stiffness of the structure, respectively, and are of the following form:

$$\mathbf{M} = \mathrm{diag}(m_i) \quad (i = 1, 2, \ldots, n_{\mathrm{r}}) \tag{4a}$$

$$\mathbf{C} = \begin{bmatrix} c_1 + c_2 & -c_2 & 0 & 0 \\ -c_2 & c_2 + c_3 & -c_3 & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & -c_{n_{\mathrm{r}}} & c_{n_{\mathrm{r}}} \end{bmatrix}, \quad \mathbf{K} = \begin{bmatrix} k_1 + k_2 & -k_2 & 0 & 0 \\ -k_2 & k_2 + k_3 & -k_3 & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & -k_{n_{\mathrm{r}}} & k_{n_{\mathrm{r}}} \end{bmatrix} \tag{4b}$$

where $m_i, c_i, k_i, (i = 1, \ldots, n_{\mathrm{r}})$ are unknown positive constant values with known bounds. Scalars m_0, c_0 and k_0 are the mass, damping and stiffness of the structural base, respectively. Equation (3) represents the internal dynamics of a hydraulic actuator's chamber, with v being the average output actuator force, u the total fluid flow rate of the actuator's chamber, P_{a} the actuator effective piston's area, C_{v} the chamber's volume, β the bulk modulus of the hydraulic fluid and C_{ℓ} the coefficient of leakage. For the mentional simplicity in the subsequent sections, v and u will be denominated as *actuator control force* and *actuator command control*, respectively. The seismic excitation is characterized by a displacement function d and its velocity $\dot{d}$. Scalar

function $f_N(q_c, \dot{q}_c, d, \dot{d}) \in \mathbb{R}$ represents an additional horizontal force produced on the structural base by non-linearities of the isolator.

Assumption 1. $f(q_c, \dot{q}_c, d, \dot{d}) \in \mathbb{R}$ is an unknown scalar function such that for some known positive continuous function $\phi(q_c, \dot{q}_c) \in \mathbb{R}$ the following relationship holds:

$$|f(q_c, \dot{q}_c, d, \dot{d})| \leqslant \phi(q_c, \dot{q}_c) \leqslant \phi_0 + \phi_1 \|\mathbf{z}\|, \quad \mathbf{z} =: [q_c, \dot{q}_c]^{\mathrm{T}}, \quad \|\mathbf{z}\| =: [q_c^2 + \dot{q}_c^2]^{1/2} \tag{5}$$

with ϕ_0 and ϕ_1 being some known non-negative constants.

Define the relative co-ordinates of the structure $(\mathbf{q}_r^*, \dot{\mathbf{q}}_r^*)$ as follows:

$$\mathbf{q}_r^* =: \mathbf{q}_r - \mathbf{J} q_c, \quad \dot{\mathbf{q}}_r^* =: \dot{\mathbf{q}}_r - \mathbf{J}\dot{q}_c, \quad \mathbf{J} =: [1, 1, \ldots, 1]^{\mathrm{T}} \tag{6}$$

then, the following results on the stability of the main structure subsystem are obtained.[3]

Proposition 1. The unforced main structure subsystem (1) *(i.e. with the coupling term $\boldsymbol{\rho}(q_c, \dot{q}_c) \equiv 0$ for $t \geqslant 0$) is globally exponentially stable for any bounded initial conditions.*

Proposition 2. (i) *If the state variables $(q_c, \dot{q}_c)$ of the base and the coupling term $\boldsymbol{\rho}(q_c, \dot{q}_c)$ are uniformly bounded, then the main structure subsystem is stable for any bounded initial conditions and the state variables $(\mathbf{q}_r^*, \dot{\mathbf{q}}_r^*)$ of the structure are bounded for all $t \geqslant 0$.* (ii) *If the state variables $(q_c, \dot{q}_c)$ of the base and the coupling term $\boldsymbol{\rho}(q_c, \dot{q}_c)$ are uniformly bounded for all $t \geqslant 0$ and converge to zero exponentially as $t \to \infty$, then the main structure subsystem is exponentially stable for any bounded initial condition and the state variables $(\mathbf{q}_r^*, \dot{\mathbf{q}}_r^*)$ of the structure converge to zero exponentially as $t \to \infty$.*

3. ROBUST CONTROL STRATEGIES

The objective of robust control design is to drive the state variables $(q_c, \dot{q}_c)$ of the base and $(\mathbf{q}_r^*, \dot{\mathbf{q}}_r^*)$ of the structure to their zero equilibrium positions in the presence of unknown seismic excitation, parametrical uncertainties and actuator dynamics. In the control design, the base isolated structure with actuator dynamics [equations (1)–(3)] is regarded as being formed by two cascade loops: the structure loop [i.e. the main structure subsystem (1) plus the base isolation subsystem (2)] and the actuator loop [i.e. the actuator subsystem (3)]. Unlike the dynamic model without actuator dynamics, the virtual actuator control force v in equation (2) cannot be synthesized directly. Instead, v is the output of a first-order actuator loop [equation (3)]. In this paper, the design procedure is organized as a two-step procedure in accordance with the composite control strategy. Firstly, in the structure loop, v is regarded as a control variable for the subsystems (1) and (2) and a 'desired' actuator control force v_d will be designed to make the base isolated structure without actuator dynamics to be exponentially stable. Secondly, in the actuator loop, a robust command controller u will be designed such that the 'real' actuator control force v tracks exponentially the 'desired' actuator control force v_d and thus the global exponential stability is achieved in the base isolated structure with actuator dynamics. The overall closed-loop control system is shown in Figure 1.

3.1. Design of the 'desired' actuator control force v_d

Now, a 'desired' actuator control force v_d is designed such that the control objective is achieved for the base isolated structure without actuator dynamics. A robust control scheme based on the sliding-mode principle[9] is chosen to drive the state variables $(q_c, \dot{q}_c)$ of the base to zero exponentially so that the coupling term $\boldsymbol{\rho}(q_c, \dot{q}_c)$ between the main structure subsystem (1) and the base isolation subsystem (2) vanishes

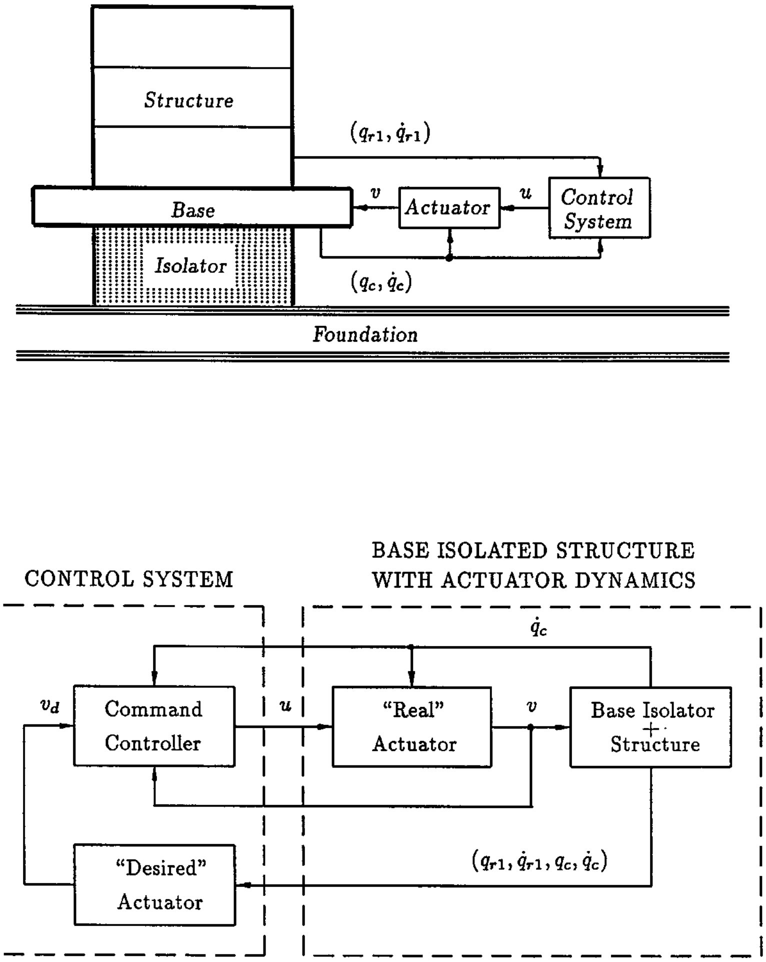

Figure 1. Overall closed-loop control system

exponentially. Thus, the state variables $(\mathbf{q}_r^*, \dot{\mathbf{q}}_r^*)$ of the structure converge to zero asymptotically according to Proposition 2.

Define a sliding function $s \in \mathbb{R}$ for the isolation subsystem (2):

$$s = \dot{q}_c + \delta q_c \tag{7}$$

where δ is a positive scalar being related to the stability of the base isolation subsystem (2) in sliding mode. Under Assumption 1 and by using only the state variables $(q_c, \dot{q}_c)$ of the base and $(q_{r1}, \dot{q}_{r1})$ of the first floor as feedback information, the following robust control law is proposed for the generation of a 'desired' actuator control force v_d in the base isolation subsystem (2):

$$v_d = -\Lambda s - \psi_s \frac{s}{|s| + \theta e^{-\gamma t}} - \psi_x \frac{\|\mathbf{x}\|^2 s}{\|\mathbf{x}s\| + \theta e^{-\gamma t}} \tag{8}$$

$$\mathbf{x} = [q_c, \dot{q}_c, q_{r1}, \dot{q}_{r1}]^T, \quad \|\mathbf{x}\| =: [q_c^2 + \dot{q}_c^2 + q_{r1}^2 + \dot{q}_{r1}^2]^{1/2}, \quad \|\mathbf{x}s\| =: [(q_c^2 + \dot{q}_c^2 + q_{r1}^2 + \dot{q}_{r1}^2)s^2]^{1/2} \tag{9a}$$

$$\psi_s \geqslant \phi_0, \quad \psi_x \geqslant \phi_1 + \phi_2, \quad \phi_2 =: \mathrm{Max}(k_0 + 2k_1 + c_1 + |c_0 + c_1 - \delta m_0|) \tag{9b}$$

with ϕ_0, ϕ_1, $\mathbf{z}$ and $\|\mathbf{z}\|$ being defined by equation (5), Λ, θ and γ being positive constants. The following result is concerned with the global exponential stability of the base isolation subsystem (2).

Theorem 1. Under Assumptions 1 *and by using the 'desired' actuator control force v_d as defined by equations* (8) *and* (9), *the sliding function s and the state variables $(q_c, \dot{q}_c)$ of the base are bounded for all $t \geqslant 0$ and converge to zero exponentially as $t \to \infty$ for any bounded initial conditions.*

Proof. Consider a Lyapunov function candidate

$$V_1 =: \tfrac{1}{2} m_0 s^2 + \delta \Lambda q_c^2 \tag{10}$$

By differentiating V_1 with respect to time t and using equations (2), (8) and (9), one has

$$\begin{aligned}
\dot{V}_1 &= m_0 s\dot{s} + 2\delta\Lambda q_c \dot{q}_c = -\Lambda s^2 + 2\delta\Lambda q_c s - 2\delta^2\Lambda q_c^2 - (q_c, \dot{q}_c, d, \dot{d})s \\
&\quad - [(c_0 + c_1 - \delta m_0)\dot{q}_c + (k_0 + k_1)q_c - c_1\dot{q}_{r1} - k_1 q_{r1}]s - \psi_s \frac{s^2}{|s| + \theta e^{-\gamma t}} - \psi_x \frac{\|\mathbf{x}s\|^2}{\|\mathbf{x}s\| + \theta e^{-\gamma t}} \\
&\leqslant -\Lambda s^2 + 2\delta\Lambda q_c s - 2\delta^2\Lambda q_c^2 + \phi_0|s| + \phi_1\|\mathbf{z}s\| + \phi_2\|\mathbf{x}s\| - \psi_s \frac{s^2}{|s| + \theta e^{-\gamma t}} - \psi_x \frac{\|\mathbf{x}s\|^2}{\|\mathbf{x}s\| + \theta e^{-\gamma t}} \\
&\leqslant -\Lambda s^2 + 2\delta\Lambda q_c s - 2\delta^2\Lambda q_c^2 + \phi_0 \left\{|s| - \frac{s^2}{|s| + \theta e^{-\gamma t}}\right\} + (\phi_1 + \phi_2)\left\{\|\mathbf{x}s\| - \frac{\|\mathbf{x}s\|^2}{\|\mathbf{x}s\| + \theta e^{-\gamma t}}\right\} \\
&\leqslant -[s \;\; q_c]\begin{bmatrix} \Lambda & \delta\Lambda \\ \Lambda\delta & 2\delta^2\Lambda \end{bmatrix}\begin{bmatrix} s \\ q_c \end{bmatrix} + \eta\theta e^{-\gamma t}, \quad \eta =: \phi_0 + \phi_1 + \phi_2
\end{aligned} \tag{11}$$

Define

$$\alpha =: \lambda_{\min}(\mathbf{Q}_1)/\lambda_{\max}(\mathbf{P}_1), \quad \|[s(\cdot), q_c(\cdot)]^{\mathrm{T}}\| =: [s^2(\cdot) + q_c^2(\cdot)]^{1/2} \tag{12}$$

$$\mathbf{P}_1 =: \begin{bmatrix} m_0/2 & 0 \\ 0 & \delta\Lambda \end{bmatrix}, \quad \mathbf{Q}_1 =: \begin{bmatrix} \Lambda & \delta\Lambda \\ \Lambda\delta & 2\delta^2\Lambda \end{bmatrix} \tag{13}$$

where $\lambda_{\min}(\cdot)$ and $\lambda_{\max}(\cdot)$ are the minimum eigenvalue and the maximum eigenvalue of the matrix $(\cdot)$, respectively. $\mathbf{P}_1 = \mathbf{P}_1^{\mathrm{T}}$ and $\mathbf{Q}_1 = \mathbf{Q}_1^{\mathrm{T}}$ are positive-definite real matrices. Since the first term on the right-hand side of equation (11) is negative definite, one gets

$$\left\|\begin{bmatrix} s(t) \\ q_c(t) \end{bmatrix}\right\| \leqslant \left\{\frac{\lambda_{\max}(\mathbf{P}_1)}{\lambda_{\min}(\mathbf{P}_1)}\left\|\begin{bmatrix} s(0) \\ q_c(0) \end{bmatrix}\right\|^2 + \frac{\eta\theta t}{\lambda_{\min}(\mathbf{P}_1)}\right\}^{1/2} e^{-\alpha t/2} \quad \textit{if} \quad \alpha = \gamma \tag{14}$$

$$\left\|\begin{bmatrix} s(t) \\ q_c(t) \end{bmatrix}\right\| \leqslant \left\{\frac{\lambda_{\max}(\mathbf{P}_1)}{\lambda_{\min}(\mathbf{P}_1)}\left\|\begin{bmatrix} s(0) \\ q_c(0) \end{bmatrix}\right\|^2 e^{-\alpha t} + \frac{\eta\theta(e^{-\gamma t} - e^{-\alpha t})}{\lambda_{\min}(\mathbf{P}_1)(\alpha - \gamma)}\right\}^{1/2} \quad \textit{if} \quad \alpha \neq \gamma \tag{15}$$

It is known from equations (14) and (15) and (7) that the sliding function s and the state variables $(q_c, \dot{q}_c)$ of the base are bounded for all $t \geqslant 0$ and converge to zero exponentially as $t \to \infty$ for any bounded initial conditions.

3.2. Design of the command control u

Now, a command control law u is designed for the base isolated structure with actuator dynamics such that the 'real' actuator control force v tracks exponentially the 'desired' actuator control force v_d obtained in

the above section. Thus, the global exponential stability is achieved in the overall base isolated structure with actuator dynamics. The following two cases will be studied:

The parameters of the actuator subsystem are known constants.
The parameters of the actuator subsystem are unknown constants but with known upper bounds.

3.2.1. Actuator with known parameters. Denote

$$\tilde{v} =: v - v_{\mathrm{d}} \tag{16}$$

Suppose that the actuator parameters P_{v}, P_{ℓ} and P_{a} are known constants. The following command control law is adopted:

$$u = P_{\mathrm{v}}\dot{v}_{\mathrm{d}} + P_{\ell}v_{\mathrm{d}} + P_{\mathrm{a}}\dot{q}_{\mathrm{c}} \tag{17}$$

with v_{d} being defined by equations (8) and (9). A direct extension of Theorem 1 for the control law (17) is given below.

Theorem 2. By applying the command control law (17) *to the base isolated structure with actuator dynamics* [*equations* (1)–(3)], *the sliding function s and the state variables* $(q_{\mathrm{c}}, \dot{q}_{\mathrm{c}})$ *of the base are bounded for all* $t \geqslant 0$ *and converge to zero exponentially as* $t \to \infty$ *for any bounded initial conditions.*

Proof. Consider a Lyapunov function candidate

$$V =: V_1 + V_2, \quad V_2 =: \tfrac{1}{2}P_{\mathrm{v}}\tilde{v}^2 \tag{18}$$

where $V_1(t)$ is defined by equation (10). By differentiating V_2 with respect to time t and using equations (3), (16) and (17), one has

$$\dot{V}_2 = -P_{\ell}\tilde{v}^2 \tag{19}$$

thus, using equations (11) and (19), one gets

$$\dot{V} = \dot{V}_1 + \dot{V}_2 \leqslant -[s\ \ q_{\mathrm{c}}\ \ \tilde{v}]\begin{bmatrix} \Lambda & \delta\Lambda & 0 \\ \Lambda\delta & 2\delta^2\Lambda & 0 \\ 0 & 0 & P_{\ell} \end{bmatrix}\begin{bmatrix} s \\ q_{\mathrm{c}} \\ \tilde{v} \end{bmatrix} + \eta\theta\mathrm{e}^{-\gamma t} \tag{20}$$

Define

$$\omega =: \lambda_{\min}(\mathbf{Q}_2)/\lambda_{\max}(\mathbf{P}_2), \quad \|[s(\cdot), q_{\mathrm{c}}(\cdot), \tilde{v}(\cdot)]^{\mathrm{T}}\| = [s^2(\cdot) + q_{\mathrm{c}}^2(\cdot) + \tilde{v}^2(\cdot)]^{1/2} \tag{21}$$

$$\mathbf{P}_2 =: \begin{bmatrix} m_0/2 & 0 & 0 \\ 0 & \delta\Lambda & 0 \\ 0 & 0 & P_{\mathrm{v}}/2 \end{bmatrix}, \quad \mathbf{Q}_2 =: \begin{bmatrix} \Lambda & \delta\Lambda & 0 \\ \Lambda\delta & 2\delta^2\Lambda & 0 \\ 0 & 0 & P_{\ell} \end{bmatrix} \tag{22}$$

where $\mathbf{P}_2 = \mathbf{P}_2^{\mathrm{T}}$ and $\mathbf{Q}_2 = \mathbf{Q}_2^{\mathrm{T}}$ are positive definite real matrices. By adopting the similar procedure as that in the proof of Theorem 1, one has

$$\left\|\begin{bmatrix} s(t) \\ q_{\mathrm{c}}(t) \\ \tilde{v}(t) \end{bmatrix}\right\| \leqslant \left\{\frac{\lambda_{\max}(\mathbf{P}_2)}{\lambda_{\min}(\mathbf{P}_2)}\left\|\begin{bmatrix} s(0) \\ q_{\mathrm{c}}(0) \\ \tilde{v}(0) \end{bmatrix}\right\|^2 + \frac{\eta\theta t}{\lambda_{\min}(\mathbf{P}_2)}\right\}^{1/2} \mathrm{e}^{-\omega t/2} \quad \text{if} \quad \omega = \gamma \tag{23}$$

$$\left\|\begin{bmatrix} s(t) \\ q_{\mathrm{c}}(t) \\ \tilde{v}(t) \end{bmatrix}\right\| \leqslant \left\{\frac{\lambda_{\max}(\mathbf{P}_2)}{\lambda_{\min}(\mathbf{P}_2)}\left\|\begin{bmatrix} s(0) \\ q_{\mathrm{c}}(0) \\ \tilde{v}(0) \end{bmatrix}\right\|^2 \mathrm{e}^{-\omega t} + \frac{\eta\theta(\mathrm{e}^{-\gamma} - \mathrm{e}^{-\omega t})}{\lambda_{\min}(\mathbf{P}_2)(\omega - \gamma)}\right\}^{1/2} \quad \text{if} \quad \omega \neq \gamma \tag{24}$$

It is known from equations (23), (24) and (7) that by using the command control law (17) the 'real' actuator control force v tracks exponentially the 'desired' actuator control force v_d such that the sliding function s and the state variables $(q_c, \dot{q}_c)$ of the base are bounded for all $t \geqslant 0$ and converge to zero exponentially as $t \to \infty$ for any bounded initial conditions.

Corollary 2.1. According to Theorem 2, *Proposition* 2 *and equation* (6), *the state variables* $(\mathbf{q}_r^*, \dot{\mathbf{q}}_r^*)$ *of the structure are bounded for all* $t \geqslant 0$ *and converge to zero exponentially as* $t \to \infty$ *for any bounded initial conditions.*

3.2.2. Actuator with unknown parameters. Now, suppose that the actuator parameters P_v, P_ℓ and P_a are unknown constants, but with known upper bounds which satisfy the following relationships:

$$|P_v - \bar{P}_v| \leqslant \sigma_v, \quad |P_\ell - \bar{P}_\ell| \leqslant \sigma_\ell, \quad P_a - \bar{P}_a| \leqslant \sigma_a \tag{25}$$

where $\bar{P}_v$, $\bar{P}_\ell$, $\bar{P}_a$, σ_v, σ_ℓ and σ_a are some known positive constants. In this case, it is assumed that the 'real' actuator control force v is measurable. Then, the following command control law is proposed:

$$u = \bar{P}_v \dot{v}_d + \bar{P}_\ell v_d + \bar{P}_a \dot{q}_c - \sigma_v |\dot{v}_d|^2 \Omega_v - \sigma_\ell |v_d|^2 \Omega_\ell - \sigma_a |\dot{q}_c|^2 \Omega_a \tag{26}$$

where

$$\Omega_v =: \frac{\tilde{v}}{|\tilde{v}||\dot{v}_d| + \theta e^{-\gamma t}}, \quad \Omega_\ell =: \frac{\tilde{v}}{|\tilde{v}||v_d| + \theta e^{-\gamma t}}, \quad \Omega_a =: \frac{\tilde{v}}{|\tilde{v}||\dot{q}_c| + \theta e^{-\gamma t}} \tag{27}$$

with v_d and $\tilde{v}$ being defined by equation (8) and (16), respectively.

Theorem 3. *By applying the command control laws* (26) *and* (27) *to the base isolated structure with actuator dynamics* [*equations* (1)–(3)], *the sliding function s and the state variables* $(q_c, \dot{q}_c)$ *of the base are bounded for all* $t \geqslant 0$ *and converge to zero exponentially as* $t \to \infty$ *for any bounded initial conditions.*

Proof. Consider a Lyapunov function candidate

$$V =: V_1 + V_2, \quad V_2 =: \tfrac{1}{2} P_v \tilde{v}^2 \tag{28}$$

where $V_1(t)$ is defined by equation (10). By differentiating V_2 with respect to time t and using equation (3), (25) and (27), one has

$$\begin{aligned} \dot{V}_2 &= -P_\ell \tilde{v}^2 + [(\bar{P}_v - P_v)\dot{v}_d + (\bar{P}_\ell - P_\ell)v_d + (\bar{P}_a - P_a)\dot{q}_c - \sigma_v|\dot{v}_d|^2\Omega_v - \sigma_\ell|v_d|^2\Omega_\ell - \sigma_a|\dot{q}_c|^2\Omega_a]\tilde{v} \\ &\leqslant -P_\ell\tilde{v}^2 + [\sigma_v|\dot{v}_d|(1 - |\dot{v}_d||\Omega_v|) + \sigma_\ell|v_d|(1 - |v_d||\Omega_\ell|) + \sigma_a|\dot{q}_c|(1 - |\dot{q}_c||\Omega_a|)]\tilde{v} \leqslant -P_\ell\tilde{v}^2 + \theta e^{-\gamma t} \end{aligned} \tag{29}$$

where

$$\sigma =: \sigma_v + \sigma_\ell + \sigma_a \tag{30}$$

thus, using equations (11) and (29), one gets

$$\dot{V} = \dot{V}_1 + \dot{V}_2 \leqslant -[s \; q_c \; \tilde{v}] \begin{bmatrix} \Lambda & \delta\Lambda & 0 \\ \Lambda\delta & 2\delta^2\Lambda & 0 \\ 0 & 0 & P_\ell \end{bmatrix} \begin{bmatrix} s \\ q_c \\ \tilde{v} \end{bmatrix} + (\eta + \sigma)\theta e^{-\gamma t} \tag{31}$$

By adopting the similar procedure as that in the proof of Theorem 1, one has

$$\left\| \begin{bmatrix} s(t) \\ q_c(t) \\ \tilde{v}(t) \end{bmatrix} \right\| \leqslant \left\{ \frac{\lambda_{\max}(\mathbf{P}_2)}{\lambda_{\min}(\mathbf{P}_2)} \left\| \begin{bmatrix} s(0) \\ q_c(0) \\ \tilde{v}(0) \end{bmatrix} \right\|^2 + \frac{(\eta + \sigma)\theta t}{\lambda_{\min}(\mathbf{P}_2)} \right\}^{1/2} \mathrm{e}^{-\omega t/2} \quad \text{if} \quad \omega = \gamma \tag{32}$$

$$\left\| \begin{bmatrix} s(t) \\ q_c(t) \\ \tilde{v}(t) \end{bmatrix} \right\| \leqslant \left\{ \frac{\lambda_{\max}(\mathbf{P}_2)}{\lambda_{\min}(\mathbf{P}_2)} \left\| \begin{bmatrix} s(0) \\ q_c(0) \\ \tilde{v}(0) \end{bmatrix} \right\|^2 \mathrm{e}^{-\omega t} + \frac{(\eta + \sigma)\theta(\mathrm{e}^{-\gamma t} - \mathrm{e}^{-\omega t})}{\lambda_{\min}(\mathbf{P}_2)(\omega - \gamma)} \right\}^{1/2} \quad \text{if} \quad \omega \neq \gamma \tag{33}$$

where the definition of ω, $\mathbf{P}_2$ and $\mathbf{Q}_2$ is the same as that in the proof of Theorem 2. It is known from equation (32) and (33) that by using the command control law (26) and (27) the 'real' actuator control force v tracks exponentially the 'desired' actuator control force v_d in the presence of parametrical uncertainties in the actuator subsystem (3) such that the sliding function s and the state variables $(q_c, \dot{q}_c)$ of the base are bounded for all $t \geqslant 0$ and converge to zero exponentially as $t \to \infty$ for any bounded initial conditions.

Corollary 3.1. According to Theorem 3, *Proposition* 2 *and equation* (6), *the state variables* $(\mathbf{q}_r^*, \dot{\mathbf{q}}_r^*)$ *of the structure are bounded for all* $t \geqslant 0$ *and converge to zero exponentially as* $t \to \infty$ *for any bounded initial conditions.*

4. IMPLEMENTATION ISSUES

In the preceding sections, two robust control laws have been derived such that the absolute co-ordinates $(q_c, \dot{q}_c)$ of the base and the relative co-ordinate $(\mathbf{q}_r^*, \dot{\mathbf{q}}_r^*)$ of the structure go to their zero equilibrium positions in the presence of unknown seismic excitation, parametrical uncertainties and actuator dynamics. In both control laws, the desired actuator control force v_d as defined in equations (8) and (9) has been computed. Notice that the design of v_d requires to select parameters ψ_s, ψ_x, Λ, θ and γ. In particular, parameters ψ_s and ψ_x are related to the model uncertainties characterized by an unknown function f which accomplishes the relationship (5) related to ϕ_0 and ϕ_1. The parameters ψ_x is also related to ϕ_2 defined in equation (9b), which is related to the bounds on structural parameters. Parameters Λ, θ and γ are positive constants chosen by the designer to obtain the desired transient and steady dynamics of the closed-loop systems.

Notice that the computation of the desired actuator control force v_d requires the use of q_c, $\dot{q}_c$, q_{r_1} and $\dot{q}_{r_1}$, which are absolute displacements and velocities of the base and the first floor, as feedback signals. In fact, measuring the absolute motion of the base and the structure may be considered as an interesting practical issue of the actual implementation of those control laws developed by using the model of absolute co-ordinates, although it has not been the main concern of this paper. About the possibility of sensing or measuring the absolute motion of the structure, some, important results have been obtained recently.[10]

In the first roboust command control law defined in equation (17), it is assumed that the actuator parameters are known. The absolute acceleration of the base $\ddot{q}_c$ and that of the first floor $\ddot{q}_{r_1}$ have to be measured and then used in the computation of $\dot{v}_d$, which is needed for the implementation of the robust command control law (17). In the second robust command control law defined in equations (26) and (27), it is assumed that the actuator parameters are unknown and bounded with known bounds as expressed in equation (25). In this case, more computation efforts have to be made for the implementation of the robust command control laws (26) and (27). Notice that the actual value of actuator control force v has to be measured and used as an additional feedback information so that the paucity of knowledge on the system can be compensated for.

5. NUMERICAL EXAMPLE

Consider a ten-storeyed base isolated building structure described by equations (1)–(3). The mass of each floor, including that of the base, is 6×10^5 kg. The stiffness of the base is $1{\cdot}184\times 10^7$ N/m and its damping ratio is 0·1. The stiffness of the structure varies in 5×10^7 N/m between floors, from 9×10^8 N/m, for the first one to $4{\cdot}5\times 10^8$ N/m, for the top one with the damping ratio being 0·05. The actuator dynamics is described by equation (3) with $C_v = 1{\cdot}518\times 10^{-3}\,\mathrm{m}^3$, $\beta = 2{\cdot}1\times 10^3\,\mathrm{MN/m}^2$, $C_1 = 1{\cdot}0\times 10^{-6}\,\mathrm{m}^5/\mathrm{N\,s}$ and $P_a = 5{\cdot}06\times 10^{-2}\,\mathrm{m}^2$. A frictional device is used for the base isolation, so that the non-linear force f_N is described by the following equation:

$$f_N(q_c, \dot{q}_c, d, v) = -\,\mathrm{sgn}(x)[\mu_{\max} - \Delta\mu e^{-v|\dot{x}|}]\mathbf{Q} \tag{34}$$

where $x = q_c - d$; $\mathbf{Q}$ is the force normal to the friction surface; $\mu = 0{\cdot}1$ is the friction coefficient; $v = 2{\cdot}0$; $\mu_{\max} = 0{\cdot}185$ is the coefficient for high sliding velocity; $\Delta\mu = 0{\cdot}09$ is the difference between $\mu_{\max}$ and the friction coefficient for low sliding velocity. In the simulation, the seismic excitation has been that of the *El Centro* (1940) earthquake as shown in Figure 6. Then, the following relationship holds:

$$|f(q_c, \dot{q}_c, d, v)| = |-c_0 v - k_0 d + f_N(q_c, \dot{q}_c, d, v)| \leqslant \phi(q_c, \dot{q}_c) = \phi_0 \tag{35}$$

where $\phi_0 = 3{\cdot}7\times 10^6$ and $\phi_1 = 0$. The composite robust control laws (17) and (7) are used with $\delta = 387{\cdot}3$, $\Lambda = 10$, $\theta = \gamma = 0{\cdot}1$, $\phi_s = 3{\cdot}7\times 10^6$ and $\psi_x = 2{\cdot}1\times 10^9$. Both passive case (pure base isolation) and hybrid case (base isolation plus active robust control) are studied. In Figures 2–5, the time histories of the absolute

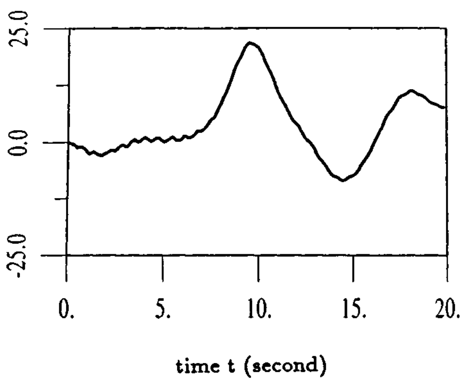

Figure 2. Absolute base displacement (cm) (passive case)

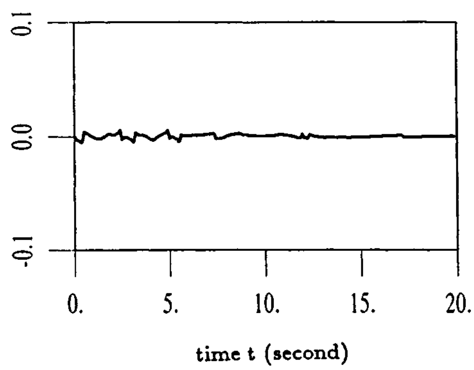

Figure 3. Absolute base displacement (cm) (hybrid case)

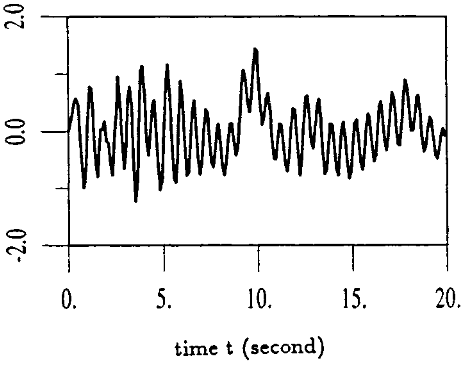

Figure 4. Relative displacement of top floor (cm) (passive case)

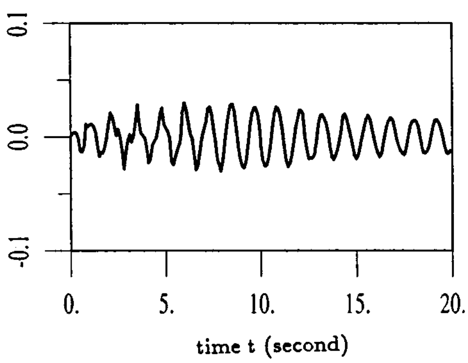

Figure 5. Relative displacement of top floor (cm) (hybrid case)

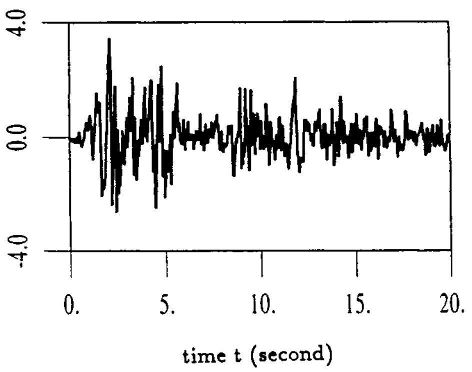

Figure 6. *El Centro* earthquake ground acceleration (m/s^2)

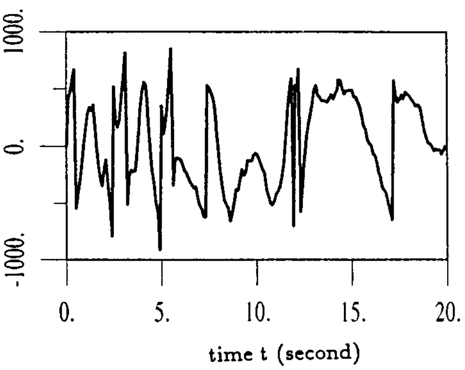

Figure 7. Actuator control force (kN) (hybrid case)

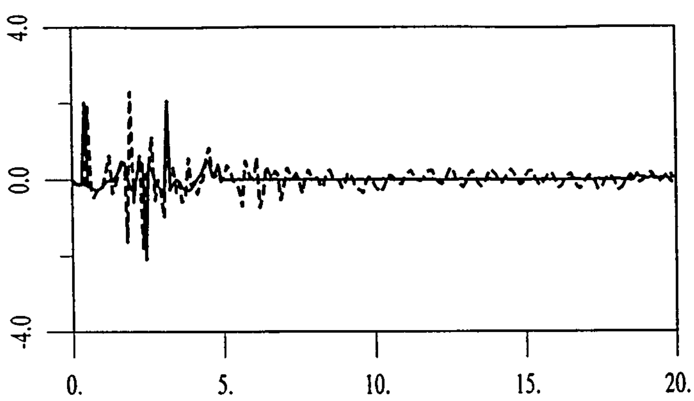

Figure 8. Absolute acceleration of the base (m/s^2) passive case (discontinuous line) and hybrid case (continuous line)

displacement of the base and the relative displacement of the top floor are shown, respectively. The actuator control force v is plotted in Figure 7. The absolute acceleration of the base and the structure are shown in Figures 8 and 9, respectively. It is seen from the simulation results that by using the proposed robust active control schemes the absolute displacement of the base, the relative displacements and the absolute accelerations of the structure have been significantly reduced and the steady dynamics of the absolute base acceleration has been improved as compared with the purely passive case. The supplied control force is reasonable relatively to the mass of base, where it is applied.

6. CONCLUSIONS

In this paper, two composite robust active control schemes have been proposed for a class of non-linear base isolated structures in the presence of unknown seismic excitation, parametrical uncertainties and actuator dynamics. Only the information on the state variables of the base and the first floor has been used in the control design. It is shown by a numerical simulation for a ten-storeyed base isolated structure under the *El Centro* earthquake that the absolute displacement of the base, the relative displacements and the absolute accelerations of the structure under the seismic excitation have been significantly reduced by using the proposed composite robust active control schemes, as compared to the purely passive case.

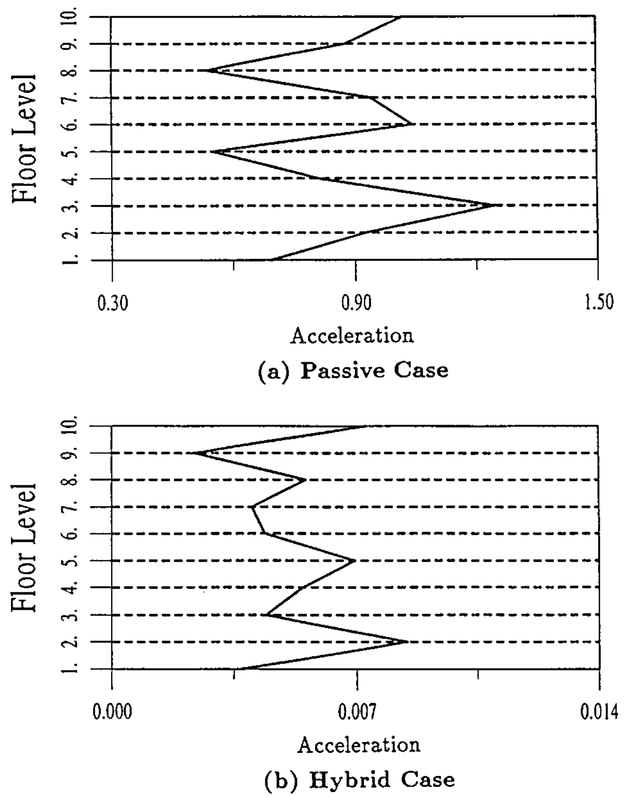

Figure 9. Absolute acceleration of the structure (m/s^2)

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